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Distinguishing the opponents promotes cooperation in well-mixed populations

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Cooperation has been widely studied when an individual strategy is adopted against all coplayers. In this context, some extra mechanisms, such as punishment, reward, memory, and network reciprocity must be introduced in order to keep cooperators alive. Here, we adopt a different point of view. We study the adoption of different strategies against different opponents instead of adoption of the same strategy against all of them. In the context of the prisoner dilemma, we consider an evolutionary process in which strategies that provide more benefits are imitated and the players replace the strategy used in one of the interactions furnishing the worst payoff. Individuals are set in a well-mixed population, so that network reciprocity effect is excluded and both synchronous and asynchronous updates are analyzed. As a consequence of the replacement rule, we show that mutual cooperation is never destroyed and the initial fraction of mutual cooperation is a lower bound for the level of cooperation. We show by simulation and mean-field analysis that (i) cooperation dominates for synchronous update and (ii) only the initial mutual cooperation is maintained for asynchronous update. As a side effect of the replacement rule, an "implicit punishment" mechanism comes up in a way that exploitations are always neutralized providing evolutionary stability for cooperation.

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I. INTRODUCTION

Cooperative dilemma was initially studied in the framework of classical game theory. Usually individuals have two strategies: cooperation and defection. A cooperator provides a benefit to the opponent and pays a cost for that. A defector receives the benefits if the opponent is a cooperator. This defines a material payoff. If individuals maximize their material payoff, it is well known that defection will dominate [1]. Departing from these initial ideas, evolutionary game theory has emerged and strategy evolution in populations was studied. In this approach it is implicitly assumed the principle of natural selection, where the payoff is equated to fitness and the fittest strategy survives [2]. Following the same ideas, cultural evolution of strategies can also be defined [3]. In cultural evolution, the strategies that provide more benefits can spread due to some learning processes, like imitation process [3]. In both biological and cultural processes, it was shown that the classical theory is recovered in the replicator equation, where population is considered to be well mixed, that is, a population where everybody interacts with everybody [2].

Since in the classical static approach and in the replicator dynamics cooperators cannot survive, cooperation can only be supported with extra mechanisms [4]. Essentially two actions must take place for cooperation survival: maintenance of mutual cooperation and prevention from exploitation [3]. Cooperators can be better off only if they meet each other, so that their profits exceed defectors' profits. If the individuals perceive that it is important what the opponents are doing in order to attend these two essential actions, reciprocal preferences can come up [5–8]. Reciprocity means that what an individual does depends on what others do to it directly or indirectly. Direct reciprocity means that I choose what to do against you depending on what you do to me. Indirect reciprocity means that my behavior toward you also depends on what you do to others. One of the simplest strategies that can perform reciprocity is "tit for tat" (TFT) strategy, where the players do whatever the coplayer did in the previous round. This simple strategy was first proposed in Axelrod's tournaments, where TFT was the greatest winner [9,10]. Another subtle way of reciprocity is network reciprocity. Individuals are set on the vertices of a network and interact only with their neighbors. In this context, cooperators form clusters of mutual cooperation and this mutualism is viewed as reciprocity [11–18]. But in the case of human behavior, the outcomes are not so simple: individuals can adopt reciprocal strategies but, motivated by internal emotion, like anger against exploitation [19], they can punish defectors [19–21]. This would not be so intriguing, as it is just another way of reciprocal motives. But the important feature is that individuals usually input costs to defectors at their own expenses. This behavior is called altruistic punishment, because individuals pay a cost to punish even if they never met the punished opponent again and because the punishment acts to weaken the defectors and the entire population gets better off [19]. Reputation, rewards, or repeated interaction, as internal motives, all interact with punishment motives [5,6]. Punishment involves some subtle questions and gives rise to another evolutionary puzzle: altruistic punishment, although seemingly usual in humans, may be a maladaptive trait as the punishers get worse payoffs [8].

There are other mechanisms that can sustain cooperation [4], but the great part of them assumes that individuals cannot adopt different strategies against their opponents. In these approaches, the same strategy is adopted against all of the possible coplayers. The possibility of recognizing the partner might be not reasonable in animal societies, because for animals even the recognition of a cooperative individual is hard [22], being even harder to remember every opponent. But in human society, like in partner marketplaces, this is

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reasonable, and it has been assumed in models of reputation [7,23]. In these models individuals must be able to recognize the reputation of the others, and they act according to some internal preferences that drive punishment or rewarding. But what if the individuals just keep track of what the others are doing and try to improve its own profits with no assumption of internal desire to punish or to reward? This can be accomplished by the adoption of different strategies against different opponents and it was studied in the context of network reciprocity with synchronous update [24]. Instead of playing the same strategy against all of the neighbors, individuals can choose a different strategy against each opponent. If each player updates its strategies by possibly imitating a successful random neighbor and replaces the strategy in the interaction that gives the worst payoff, it was shown for square lattices that cooperation was strongly supported, even for huge defection tendency. Moreover, it was shown that these results are robust against misjudgments of the worst interaction. The possibility of opponent differentiation introduces a mechanism of punishing without costs and without any kind of internal preferences, except the desire of maximizing the individual payoff. We call this punishment implicit punishment. But in that work [24], the possibility of adoption of different strategies was introduced in the context of network reciprocity. What happens if network reciprocity is excluded? Here, we analyze this model in well-mixed populations, which means that we are excluding network reciprocity effects. The other important feature of the model is the synchronous update assumption. It is well known that results may be striking different if asynchronous update is used [25,26]. Here, we analyze the model with both synchronous and asynchronous updates. We show that cooperation still remains alive, although for asynchronous update it achieves its lower bound level. We analyze the model using computer simulations and a mean-field technique.

II. MODEL

Let us state the model formally. We study the prisoner dilemma in a population of size N as the scenario for the cooperation problem. We consider well-mixed population, which means that each player interacts with everybody. The strategy vector of an individual is $\vec{S}=(S_1,\ldots,S_j,\ldots,S_{N-1})$, where S_j can be C (cooperation) or D (defection). So individuals are merged in $N-1$ interactions. If in one of these interactions an individual plays C against an opponent who is playing D, we denote this interaction as (C,D) (the first entry is the strategy of the focal player and the second entry is the opponent strategy). The definitions of the strategies and of the interactions are presented in Fig. 1. In a simplified version of the prisoner dilemma payoff [16], the payoff of a D strategy against a C strategy is $P(D,C)=b$, where $b>1$ is the defection tendency. Using the same notation, we have that $P(D,D)=\epsilon$, with $\epsilon\ll 1$, $P(C,C)=1$, and $P(C,D)=0$. In each round, each player interacts with all of the other players and the pairwise payoffs are summed as a cumulative payoff.

The strategies are updated according to the imitation rule with both synchronous and asynchronous updates. For synchronous update each player randomly chooses one neighbor and compares the cumulative payoffs. If the opponent cumulative payoff is bigger than its own one, it imitates the strategy that the opponent is using against it with probability p proportional to the difference of cumulative payoffs ΔP_{cum} [27], namely,

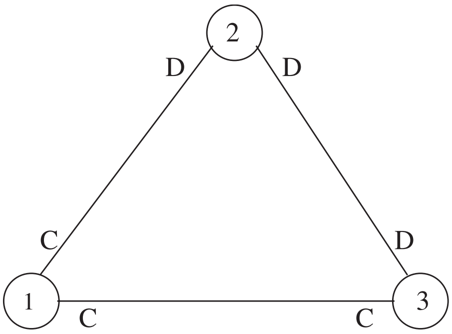

FIG. 1. Strategies and interactions for a population of size $N=3$. Player 1 has two interactions: (C,D) with player 2 and (C,C) with player 3. Player 2 has two interactions: (D,C) with player 1 and (D,D) with player 3. Player 3 has two interactions: (C,C) with player 1 and (D,D) with player 2.

$$p=|\Delta P_{cum}|/[(N-1)b].$$

On the other hand, if the opponent cumulative payoff is lower than its own one, the focal player remains with the same strategies. If imitation takes place, the new strategy replaces the strategy used in the interaction that gives the worst payoff. If there are more than one interaction with the worst pairwise payoff, one of these interactions is randomly chosen. The worst pairwise payoff of the focal player is given by the interaction (C,D), followed by (D,D), (C,C), and (D,C). This means that if the focal player has a (C,D) interaction and a defection strategy is imitated, the (C,D) interaction is replaced with (D,D). In the asynchronous update, first an individual is randomly chosen. Then it can imitate and possibly replace one of its strategies as in the synchronous case. After the individual update, the cumulative payoffs are updated, and another individual is randomly chosen to update its strategies. A time step consists of N of such processes.

III. EVOLUTIONARY ANALYSIS

The simulations were performed using networks of sizes $N=40$ and 100. The fraction of cooperation (f_c) adopted by all of the players in all of their interactions was evaluated. If n_c is the quantity of C strategies used in all of the interactions by all of the players, we have $0\leq n_c\leq N(N-1)$ and $f_c=n_c/N(N-1)$. The random initial configuration consisted of 50% of cooperation and the averages were made over 100 different initial conditions. We use $\epsilon=0.001$ and we show here only the case $b=2$, although we simulated the model also for other values of b. In fact, the b value has no effect in the simulations and in the mean-field results.

Let us state one fundamental feature of the model that is independent of whether the update is synchronous or not. We state that any interaction of type (C,C) will never be re-

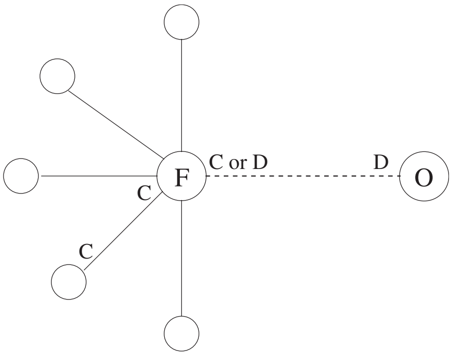

FIG. 2. The focal player F imitates a D strategy from the opponent O. Thus, the focal player has at least one (C,D) or (D,D) interaction, which is represented by the dashed line. It follows that a (C,C) interaction is never replaced, as shown in the text.

placed with (D,C). Suppose that a focal player (F in Fig. 2) imitates a defection strategy of the opponent (O in Fig. 2). This means that the focal player have at least one (C,D) or (D,D) interaction (see the dashed line in Fig. 2). These interactions furnish the payoffs 0 and ϵ that are smaller than that of a (C,C) interaction, namely, 1. It follows that (C,C) will never be replaced. This proves the existence of a lower bound for the fraction of cooperation given by the initial fraction of mutual cooperation. On the other hand, if the focal player imitates a D strategy, it will first seek for (C,D) interactions. If they are present, (C,D) will be replaced with (D,D). One can see that mutual cooperation is never destroyed, and every exploitation is punished when a defection is imitated.

In the usual game, where each player adopts a single strategy against all of its opponents, a single defector can invade a population of cooperators in an infinity well-mixed population [2]. The first remarkable feature of implicit punishment is that cooperation is evolutionary stable in well-mixed population for both synchronous and asynchronous updates. If a mutant that adopts defection against everybody appears in a population where everybody is cooperating, the mutant initially earns a huge payoff. But as soon as others imitate it, the exploited (C,D) interactions will be replaced with (D,D), neutralizing the exploitations. What is simple, but remarkable, is that the interactions that are changing are just those in which the exploiter is involved and all of the other mutual cooperation is maintained. The implicit punishment will take place until the mutant cumulative payoff is equal to a cooperator cumulative payoff. If we call n_{exp} the quantity of defection adopted by the mutant exploiter, the payoff of the mutant exploiter and the payoff of the cooperators are $P_{\text{exp}}=n_{\text{exp}}(b+\epsilon)$ and $P_{coop}=N-2$, respectively. By equating both expressions we have the equilibrium fraction given by

$$n_{\text{exp}} = \frac{N-2}{b+\epsilon}.$$

Let us now discuss the results obtained by numerical simulations for both synchronous and asynchronous updates. Initially, we set the players to cooperate with a probability of 0.5 against each one of its opponents. This gives an initial cooperation fraction of 50%. For asynchronous update, cooperation cannot dominate the population but it can coexist with defection and assumes values near the lower bound (25% for the initial condition assumed here). Figure 3 shows simulation and mean-field approximation results for the asynchronous update. For the synchronous update, cooperation dominates the entire population. Figure 4 shows the simulation and mean-field approximation results for the synchronous update. In the synchronous case, at the beginning, almost all of the exploitations are neutralized and only the initial mutual cooperation survives. After that, cooperation starts to increase very slowly until it dominates the population. The initial behavior of the cooperation fraction is shown in Fig. 5. So we can define a short-time regime and a long-time regime for the synchronous dynamics. Note that the short-time regime for the synchronous update exhibits the same behavior as the asynchronous update. The same qualitative result holds for large populations, but simulation time gets extremely huge for synchronous update. From Fig. 5 one can see that cooperation decreases in the beginning of the imitation process. But after this initial transient behavior, cooperation spreads. Figure 6 shows a finite-size analysis for the time (T) spent to reach the minimum value of the cooperation fraction before cooperation dominates in the synchronous update. Note that as along as N increases, $1/T$ goes to zero, implying that the population is wrapped in the lowest value of cooperation. On the other hand, if N is large but

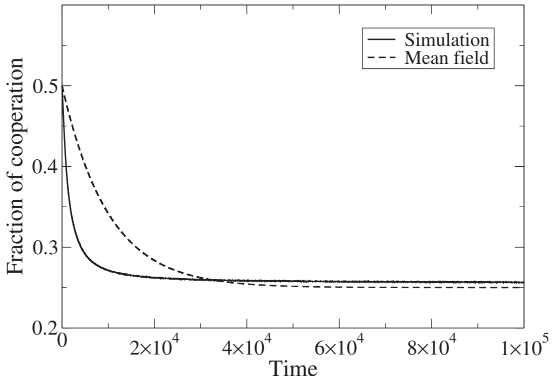

FIG. 3. Fraction of cooperation f_c for asynchronous update with $N=100$ and $b=2.0$.

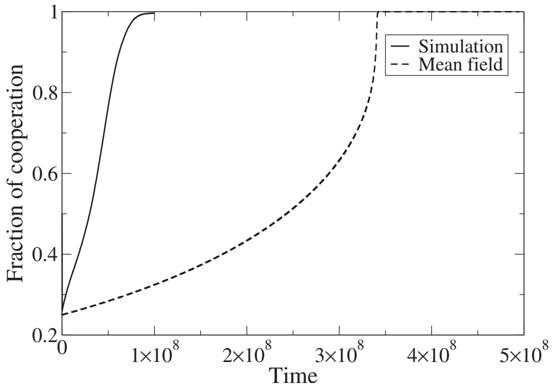

FIG. 4. Fraction of cooperation f_c for synchronous update with $N=40$ and $b=2.0$.

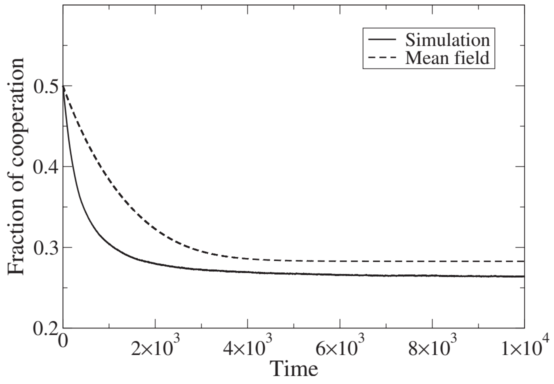

FIG. 5. Short-time regime of the cooperation fraction f_c for synchronous update with N=40 and b=2.0. The fraction of cooperation remains around the minimum value for a while and then starts to increase. The increase is not shown in this figure, but it can be seen in Fig. 4.

finite, cooperation starts to increase after a finite transient time.

Mean-field solution provides a good equilibrium analysis in well-mixed population if the usual game is considered [28]. But in structured population, it is not a good approximation [29]. Although in the present work we deal with well-mixed population, the nature of the implicit punishment model is not so simple. It is not obvious that a mean-field approach would work. So it is a remarkable result the fact that our mean-field approximation gives not only the stationary solutions, but fits reasonably the *in silico* time evolution, although for the synchronous update it fits well only in the short-time regime. Let us now derive the mean-field solution. We first analyze the asynchronous update followed by the synchronous one.

A. Mean-field approximation for the asynchronous update

Let us first define local and global interaction concentrations for a population of size $N+1$. If in an interaction a player i adopts strategy A and its opponent adopts B, where $A,B \in \{C,D\}$, we say that player i has a (A,B) interaction. Player i can have $N_{AB}(i)$ (A,B) interactions. We define the local concentration of (A,B) as the fraction of (A,B) interactions around player i, namely, $x_{AB}(i)=N_{AB}(i)/N$. For the global concentration of (A,B) we define $x_{AB}=\Sigma_i N_{AB}(i)/[(N+1)N]$. Note that $x_{CD}=x_{DC}$ and $f_c=x_{CC}+2x_{CD}$.

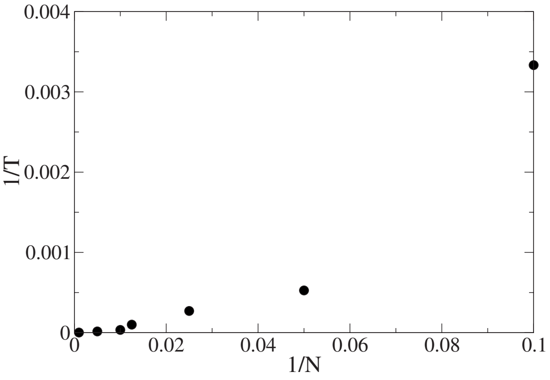

FIG. 6. The plot shows the relation $1/T \times 1/N$, where N is the population size and T is the time to reach the minimum value of cooperation for synchronous update.

We first consider a typical player that we call the focal player. We are going to study the dynamics of the local concentration of (C,D), (D,D), (C,C), and (D,C) interactions around the focal player. Let k_1, k_2, k_3, and k_4 be the quantity of such interactions around the focal player. Note that $k_1+k_2+k_3+k_4=N$. Let us assume that the probability of having k_1, k_2, k_3, and k_4 interactions is given by the respective global concentrations. The probability of a focal player configuration $\vec{k}=(k_1,k_2,k_3)$ is given by

$$\Pi(\vec{k}) = \frac{N!}{k_1!k_2!k_3!(N-k_1-k_2-k_3)!} x_{CD}^{k_1} x_{DD}^{k_2} x_{CC}^{k_3} x_{DC}^{N-k_1-k_2-k_3}.$$

We consider the other nodes as mean-field nodes. So the local concentration of (C,D), (D,D), and (C,C) interactions around those nodes is given by the configuration vector $N(x_{CD},x_{DD},x_{CC})$. Now we are going to derive the variation rate of the local concentration around the focal player. In order to set the notation, let $PF(\vec{k})$ be the cumulative payoff of the focal player when it is in a (k_1,k_2,k_3) configuration and let $PO(C,D)$ be the payoff of the opponent associated with the (C,D) focal player interaction.

Let us derive the increasing rate of the (D,D) local concentration. Just to keep the notation clear, in the interaction (D,D), the first entry refers to the focal player and the second entry refers to the opponent. So a (D,D) interaction means that both the focal player and the opponent adopt defection against each other. Suppose that the focal player is in a (k_1,k_2,k_3) configuration. For this configuration, the payoff of the focal player is given by

$$PF(\vec{k}) = (N-k_1-k_2-k_3)b + k_3 + k_2\epsilon.$$

There is just one transition that increases this quantity: (C,D) to (D,D). In this replacement, the focal player is currently adopting C and the opponent associated with the (C,D) interaction is currently adopting D. If an interaction (C,D) is replaced with (D,D), it means that the focal player imitates a D strategy from a new randomly chosen opponent. But the focal player can imitate the D strategy from (C,D) or (D,D) interactions. Let us focus on the first alternative that happens with probability k_1/N. The opponent associated with the (C,D) focal player interaction has a payoff of

$$PO(C,D) = b + (N-1)(x_{DC}b + x_{CC} + x_{DD}\epsilon).$$

The probability of imitating the D strategy from the opponent associated with the (C,D) interaction is given by

$$p = \frac{\Theta[PO(C,D) - PF(\vec{k})]}{bN},$$

where $\Theta(x)=x$ if $x>0$, and $\Theta(x)=0$, otherwise. The mean rate of increase in (D,D) by 1 unit due to the imitation from (C,D) is

$$W^{+}_{CD} = \sum_{k_1=1}^{N}\sum_{k_2=0}^{N-k_1}\sum_{k_3=0}^{N-k_1-k_2}\frac{k_1}{N}\frac{\Theta[PO(C,D)-PF(\vec{k})]}{bN}\Pi(\vec{k}).$$

Following the same lines we obtain the increasing rate of (D,D) from the other possibility [imitating a D strategy from an opponent associated with a (D,D) interaction], namely,

$$W^{+}_{DD} = \sum_{k_1=1}^{N}\sum_{k_2=0}^{N-k_1}\sum_{k_3=0}^{N-k_1-k_2}\frac{k_2}{N}\frac{\Theta[PO(D,D)-PF(\vec{k})]}{bN}\Pi(\vec{k}).$$

In this notation, the CD subscript in W^{+}_{CD} means that the defection strategy is imitated from an opponent associated with a (C,D) interaction. The DD subscript has a similar meaning.

The same analysis can be done to calculate the decreasing rate of (D,D). There is just one transition involved, namely, (D,D) to (C,D). At least one (D,D) must be present. The C strategy can be imitated from (C,C) or (D,C). But note that the focal player cannot be currently adopting a (C,D), because in this case (C,D) would give the worst payoff. So imitating a C strategy would not change the quantity of (D,D). Following the previous steps, we can define

$$\tilde{\Pi}(\vec{k}) = \frac{N!}{k_2!k_3!(N-k_2-k_3)!}x_{DD}^{k_2}x_{CC}^{k_3}x_{DC}^{N-k_2-k_3},$$

where the $\tilde{\Pi}(\vec{k})$ probability is obtained by carrying out a summation over the k_1 index of $\Pi(\vec{k})$.

So the decreasing rate of (D,D) is given by

$$W^{-}_{CC} = \sum_{k_2=1}^{N}\sum_{k_3=0}^{N-k_1}\frac{k_3}{N}\frac{\Theta[PO(C,C)-PF(\vec{k})]}{bN}\tilde{\Pi}(\vec{k})(1-\delta_{k_3,0}\delta_{k_2,N}),$$

$$W^{-}_{DC} = \sum_{k_2=1}^{N}\sum_{k_3=0}^{N-k_1}\frac{N-k_2-k_3}{N}\frac{\Theta[PO(D,C)-PF(\vec{k})]}{bN}\tilde{\Pi}(\vec{k})(1 - \delta_{k_3,0}\delta_{k_2,N}).$$

Here, we have that $\delta_{x,y}=1$ if $x=y$ and $\delta_{x,y}=0$ if $x\neq y$. The subscript CC in W^{-}_{CC} means that the cooperation strategy is imitated from an opponent associated with a (C,C) interaction. The DC subscript has a similar meaning.

Since the population is well mixed, all nodes have the same typical behavior. Thus, we can approximate the global concentrations by the local ones. The above expressions determine the variation rate of (D,D) by 1 unit. If we want the time derivative of x_{DD}, we need to divide the expressions by N and multiply by a factor of 2, because there is the contribution of the opponent update. So we have that

$$\frac{dx_{DD}}{dt} = 2\frac{1}{N}(W^{+}_{CD}+W^{+}_{DD}-W^{-}_{CC}-W^{-}_{DC}). \tag{1}$$

Following the same reasoning, one can see that x_{CC} does not change in time. Finally, as all of the mean-field variables are normalized to 1, we obtain that

$$x_{CD} = \tfrac{1}{2}(1-x_{DD}-x_{CC}).$$

We can simplify further Eq. (1) if we replace k_1, k_2, and k_3 inside the payoff expressions of the focal player with the expected value of such quantities given by the configuration probabilities, Nx_{CD}, Nx_{DD}, and Nx_{CC}, respectively. With this extra approximation the Θ function can be easily evaluated in the limit of large N and we have the following equation:

$$\frac{dx_{DD}}{dt} = 2\frac{1}{N^2}\Big\{x_{CD}+x_{DD}\frac{\epsilon}{b}[1-(1-x_{CD})^{N-1}] - \frac{x_{CC}}{b}[(1-x_{CD})^{N-1}-(x_{CC}+x_{DC})^{N-1}]\Big\}.$$

This equation can be solved numerically. Figure 3 shows the numerical and the simulation results. Note that our mean-field approximation furnishes good results when compared with *in silico* evolution. For the initial condition used here, we have that the terms inside the parentheses are 0.75 powered to $N-1$ and 0.5 powered to $N-1$. If N is large, the terms that are powered to N are very small and they can be neglected, at least for short times. This gives the following simplified equation:

$$\frac{dx_{DD}}{dt} = 2\frac{1}{N^2}\left(x_{CD}+\frac{\epsilon}{b}x_{DD}\right).$$

The solution of this equation is straightforward. We obtain that

$$x_{DD} = x^{0}_{DD} + \frac{\frac{\epsilon}{b}x^{0}_{DD}+x^{0}_{CD}}{\frac{1}{2}+\frac{\epsilon}{b}}\left\{1-\exp\left[-\frac{2}{N^2}\left(\frac{1}{2}+\frac{\epsilon}{b}\right)t\right]\right\},$$

$$x_{CC} = x^{0}_{CC},$$

$$x_{CD} = \frac{1}{2}(1-x_{DD}-x^{0}_{CC}).$$

Here the index 0 refers to the initial conditions. If we set $\epsilon=0$, just for simplicity, one can see that there is a fixed point, namely,

$$x^{\infty}_{DD} = x^{0}_{DD}+2x^{0}_{CD},$$

$$x^{\infty}_{CC} = x^{0}_{CC},$$

$$x^{\infty}_{CD} = x^{\infty}_{DC} = 0.$$

Observe that only the initial mutual cooperation can be maintained and all of the other interactions are mutual defections. Note that all of the exploitations are neutralized and that this approximation gives good results when compared to simulation data.

B. Mean-field approximation for the synchronous update

Let us treat the synchronous case. Now (C,C) can increase, because it is possible to have a (D,D) to (C,C) transition whenever two players make a (D,D) to (C,D) transition in their shared (D,D) interaction. This is an essential feature of the synchronous model. This kind of transition does not take place in asynchronous update, and that is the reason why cooperation assumes the lower bound value in the asynchronous case. We can approximate the rate of this transition by

$$\frac{dx_{CC}}{dt} = \frac{1}{Nx_{DD}}(W_{CC}^{-} + W_{DC}^{-})^2.$$

Let us explain the term in the denominator. If the focal player makes a (D,D) to (C,D) transition on a specific interaction, the mean-field player associated with this specific interaction should choose exactly this interaction. This happens with probability $1/Nx_{DD}$. If we perform the same simplifications that were already done for the asynchronous case, we have that

$$\frac{dx_{CC}}{dt} = 2\frac{1}{N^4}\left\{\frac{x_{CC}}{b}[(1-x_{CD})^{N-1} - (x_{CC}+x_{DC})^{N-1}]\right\}^2,$$

$$\frac{dx_{DD}}{dt} = 2\frac{1}{N^2}\left\{x_{CD} + x_{DD}\frac{\epsilon}{b}[1-(1-x_{CD})^{N-1}] - \frac{x_{CC}}{b}[(1-x_{CD})^{N-1} - (x_{CC}+x_{DC})^{N-1}]\right\}.$$

Figures 4 and 5 show the numerical solution of these equations. One can see from the above equations that x_{CC} increases much slower than x_{DD}. For the initial condition assumed here, x_{CC} time derivative at the beginning is almost zero, because the values inside the brackets are equal to 0.5 powered to N. But when evolution starts, great part of the (C,D) interactions is changed to (D,D) and x_{CD} is reduced to some value near zero. This makes x_{CC} to increase faster. So we have two regimes: short-time regime, when x_{CC} is kept almost constant around its initial value, and long-time regime, when x_{CC} starts to increase faster. Figures 4 and 5 show long- and short-time regimes for the evolution of fraction of cooperation. For short times, if we discard the terms that are powered to $N-1$, we have the same solution as the one obtained in the asynchronous case. This means that cooperation assumes a value near its lower bound value, given by the initial mutual cooperation. But for long-time regime, x_{CD} is near zero and the x_{CC} time derivative cannot be neglected. So x_{CC} starts to increase until it becomes equal to 1. Thus, the stationary solution for sufficient long times is

$$x_{CC}^{\infty} = 1,$$

$$x_{CD}^{\infty} = x_{DC}^{\infty} = x_{DD}^{\infty} = 0.$$

Note that for the short-time regime, shown in Fig. 5, the mean-field approximation fits well when compared to *in silico* evolution. For the long-time regime, the time evolution of the mean-field solution does not fit well, although it gives the right stationary solution.

From the above expressions and simulation data, we see that the lower value of cooperation is reached very fast in the synchronous update. But as long as the population size gets bigger, this value is reached very slowly (see Fig. 6). Besides that, if N is large, x_{CC} increases very slowly. By these reasons, for large N, in the synchronous update the population seems to be wrapped in the lower value of cooperation, although what is actually happening is that cooperation is slowly increasing, spreading until it dominates the entire population.

IV. CONCLUSION

Here, we analyzed a model that allows the individuals to choose different strategies against different opponents in well-mixed populations for both synchronous and asynchronous updates. In the context of prisoner dilemma, we showed first that cooperation is evolutionary stable for both synchronous and asynchronous updates. This means that a defector mutant cannot invade a population of cooperators. We also showed that, for an initial condition of 50% of cooperation, for synchronous update cooperation always dominates while for asynchronous update the cooperation fraction assumes the lower bound given by the initial mutual cooperation. For the synchronous update, population dynamics exhibits a short-time behavior that is similar to the asynchronous case, but for sufficient long times, cooperation spreads for large, but finite N. The crucial difference between synchronous and asynchronous updates is that in synchronous one it is possible to have a simultaneous update that allows a (D,D) to (C,C) transition. This does not happen in the asynchronous case. In a previous work, the same model was analyzed in a square lattice with synchronous update. Here, we showed that the synchronous update is crucial for cooperation dominance while network reciprocity effects are not so important. Although the asynchronous case does not provide cooperation dominance, it allows cooperation to survive at its lower bound value. Note that the result with asynchronous update is still better than the result of the usual game.

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